Being Arranged in Advance: Quantum Entanglement and Biological Feedback

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Abstract

The categorical semantics of quantum protocols proposed by Abramsky and Coecke reveals that a prearranged quantum entanglement brings a strange quantum information flow in the quantum teleportation protocol. Their formal argument leads us to the distinction between an information flow sequence and a causal sequence on the same event. If this distinction is applied to information processing biological networks, we can claim that a prearranged biological feedback can play the same role as the quantum entanglement on the emergence of a specific local structure of networks. The aim of this paper is to provide a first step toward formal arguments on changes in biology without the external time parameter.

Introduction

If something is arranged in advance and if it works well, an apparently difficult or non-intuitive event can occur. A clear example of such a phenomenon is the quantum teleportation (Bennett et al., 1993; Nielsen and Chuang, 2000), in which a prearranged entangled pair of qubits allows an arbitrary quantum bit to be transferred from one site to the other site by a classical communication. In this paper we claim that biological feedbacks also play a role in the above mentioned type of phenomena. This paper is an engagement of two recent works. One is categorical semantics of quantum protocols by Abramsky and Coecke (2004), the other is algebraic study of biological networks by the author (Haruna, 2008; Haruna and Gunji, 2008), which is also based on category theory (Mac Lane, 1971).

Abramsky and Coecke (2004) clarifies the nature of quantum information flow by recasting the standard axiomatic quantum mechanics due to von Neumann (1932). What is the most relevant to us is that their formalism enables us to distinguish between an information flow sequence and a causal sequence on the same event. Our application of their result to biological networks immediately follows from this distinction.

The problem of prearrangement is central to changes in the realm of biology since biological changes including development and evolution are in general the process of imposing new constraints on the preceding constraints (Matsuno, 1989; Salthe, 1993; Kauffman et al., 2008). Matsuno (1989) argues that changes in biology could be described as the process of equilibration toward tentative final causes. Since the propagation speed of interactions in biological systems cannot be regarded as infinite, the tentative final causes can change as equilibration proceeds. Preceding equilibration constrains and triggers a new equilibration. Salthe (1993) considers evolution and development of hierarchical systems in terms of how the lower and upper levels constrain the dynamics of the focal level of a given system. Recently Kauffman et al. (2008) regards constraints as information for biological organizations to maintain themselves and evolve. Cascades of constraints lead to changes in biological organizations.

Biological feedback will be a typical example of prearrangement in biological systems. The term 'feedback' implies that succeeding events in a system have an impact on upstream processes in the system. Hence at least a 'path' for the feedback must be prearranged so that the feedback works effectively. We do not define what is prearrangement in biological systems in general but involve it with our formal argument implicitly. In particular we consider a biological feedback in information processing biological networks.

Our previous study (Haruna, 2008; Haruna and Gunji, 2008) on biological networks considers how to describe network motifs found in information processing biological networks. Network motifs are defined as local patterns that are found in real networks significantly more often than in an ensemble of suitably prepared random networks (Milo et al., 2002). They are considered to have certain biological functions (Alon, 2006, 2007). In information processing biological networks, each node in a network is considered to be an information processing unit. The direction of an arrow in a network indicates the direction of information flow. If a pattern of information processing is specified then we can deduce that how the information processing pattern constrains the local structure of networks (Haruna, 2008; Haruna and Gunji, 2008). However, it is not yet clear that how information flows at the network level is related to a causal sequence that brings the emergence of a network motif. We show that

a simple application of the category theoretical formalism of finite-dimensional quantum mechanics by Abramsky and Coecke (2004) can reveal this problem.

In this paper category theory is the main tool to argue the formal similarity between quantum entanglement and biological feedback. We believe that the generality of category theory is sometimes helpful to reveal unexpected common structure between different areas. The argument presented in this paper would provide a concrete example of such usefulness of category theory.

This paper is organized as follows. The next section is a brief overview of the quantum teleportation protocol and its categorical description by Abramsky and Coecke. In section III we review our algebraic study of network motifs. Section IV is the main part of this paper, where we show that how a causal sequence in an information processing biological network is reconstructed by a result obtained in the categorical description of quantum mechanics. In section V we give conclusions.

Categorical description of quantum teleportation

In this section we briefly review the quantum teleportation protocol (Bennett et al., 1993) and its category theoretical description (Abramsky and Coecke, 2004). The presentation here is minimal enough for the aim of this paper. For further details see the references. See also Coecke (2004).

The quantum teleportation protocol enables one to transfer an unknown quantum state from a source A to a remote target B by only two bits classical communication between them. The protocol involves three qubits a, b and c. Initially qubit a is in a state $|\varphi\rangle$ which is a unit vector in two-dimensional complex Hilbert space $\mathcal{H} = \{\alpha|0\rangle + \beta|1\rangle|\alpha, \beta \in \mathbb{C}\}$. Qubits b and c are prearranged as an entangled state, $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, which is a unit vector in the tensor product $\mathcal{H} \otimes \mathcal{H}$. We abbreviate $|i\rangle \otimes |j\rangle$ as $|ij\rangle$ for $i, j = 0, 1$. Entangled states are defined as states that cannot be written in the form $|\phi_1\rangle \otimes |\phi_2\rangle$ for any choice of $|\phi_1\rangle$ and $|\phi_2\rangle$. Entangled states play important roles in the field of quantum information (Nielsen and Chuang, 2000).

We relocate the three qubits so that a and b are at the source A and c is at the target B. Now we perform so called *Bell-base measurement* on a and b. Each projector $P_i(i = 1, 2, 3, 4)$ associated with Bell-base measurement projects onto one of the one-dimensional subspaces spanned by the following vectors:

$$b_1 = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),\ b_2 = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),$$
$$b_3 = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),\ b_4 = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

The four outcomes of the measurement occur with equal probability, $\frac{1}{4}$. We observe the outcome of the measurement and send it from A to B. This requires classical two bits. Based on this classical information, we 'correct' the qubit c by performing one of the following unitary transformation on it:

$$U_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ U_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$U_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ U_4 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

After the unitary correction, one can see that the state of c is $|\varphi\rangle$.

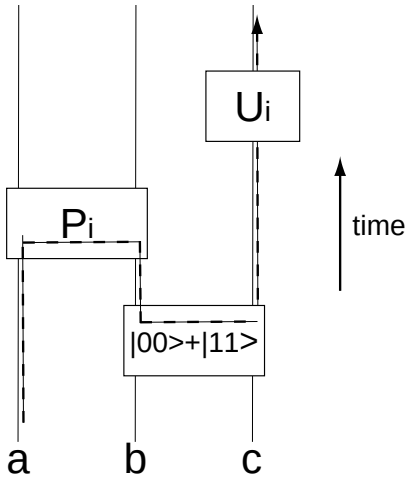

Figure 1: Quantum information flow in the teleportation protocol. The dashed arrow represents the quantum information flow.

For each observational branch, the quantum information flow seems to be 'acausal' as shown in Fig.1. Abramsky and Coecke (2004) proves that such a strange character of the quantum teleportation protocol can be captured at a more abstract level independent of the classical information flow by reformulating the finite-dimensional quantum mechanics from category theoretical point of view. A key point is that they distinguish two type of measurements appearing in the quantum teleportation protocol: one is the preparation of quantum states and the other is the indeterministic observation. These two type of measurements can be clearly distinguished by the notion of *compact closed category* (Kelly and Laplaza, 1980).

A *symmetric monoidal category* is a category $\mathcal{C}$ equipped with a *tensor product*

$$- \otimes - : \mathcal{C} \times \mathcal{C} \to \mathcal{C},$$

a *unit object* I and natural isomorphisms

$$l_A : A \cong I \otimes A,\ r_A : A \cong A \otimes I,$$
$$a_{A,B,C} : A \otimes (B \otimes C) \cong (A \otimes B) \otimes C,$$
$$s_{A,B} : A \otimes B \cong B \otimes A$$

for objects A, B, C in $\mathcal{C}$. These natural isomorphisms are required to satisfy certain coherence conditions (Mac Lane, 1971).

The definition of compact closed category by Kelly and Laplaza (1980) is as follows. A category $\mathcal{C}$ is a *compact closed category* if it is a symmetric monoidal category such that for each object A there are a dual object A^*, a *unit*

$$\eta_A : I \to A^* \otimes A$$

and a *counit*

$$\epsilon_A : A \otimes A^* \to I.$$

These data are required to satisfy the commutative diagram

$$\begin{array}{ccccc} A & \xrightarrow{r_A} & A \otimes I & \xrightarrow{1_A \otimes \eta_A} & A \otimes (A^* \otimes A) \\ {\scriptstyle 1_A}\downarrow & & & & \downarrow{\scriptstyle a_{A,A^*,A}} \\ A & \xleftarrow{l_A^{-1}} & I \otimes A & \xleftarrow{\epsilon_A \otimes 1_A} & (A \otimes A^*) \otimes A \end{array}$$

and the dual one for A^*. In other words, if a symmetric monoidal category $\mathcal{C}$ is seen as a bicategory with a single 0-cell, the 1-cells being the objects of $\mathcal{C}$ with the tensor product as their composition and the 2-cells being the morphisms of $\mathcal{C}$, the above conditions say that each object A of $\mathcal{C}$ has a right adjoint A^*. The required diagrams are 'triangular identities'.

The monoidal category of finite-dimensional vector space over a field is compact closed. This example corresponds to finite-dimensional quantum mechanics. The category of sets and relations with cartesian product $(\mathcal{R}el, \times)$ is also compact closed. This example is our main consideration in this paper. In $(\mathcal{R}el, \times)$, a one-point set $\{*\}$ is the unit object. For a set X, its dual X^* is itself, $X^* = X$. The unit for a set X is $\eta_X \subseteq \{*\} \times (X \times X)$ given by

$$\eta_X = \{(*, (x, x))|x \in X\}.$$

Similarly, the counit for X is

$$\epsilon_X = \{((x, x), *)|x \in X\}.$$

The *name* $\ulcorner f \urcorner$ and *coname* $\llcorner f \lrcorner$ of a morphism $f : A \to B$ in a compact closed category are defined by the following diagrams:

$$\begin{array}{ccc} A^* \otimes A & \xrightarrow{1_{A^*} \otimes f} & A^* \otimes B \\ {\scriptstyle \eta_A}\uparrow & & \| \\ I & \xrightarrow{\ulcorner f \urcorner} & A^* \otimes B \end{array}$$

$$\begin{array}{ccc} A \otimes B^* & \xrightarrow{\llcorner f \lrcorner} & I \\ \| & & {\scriptstyle \epsilon_B}\uparrow \\ A \otimes B^* & \xrightarrow{f \otimes 1_{B^*}} & B \otimes B^* \end{array}$$

In particular, we have $\eta_A = \ulcorner 1_A \urcorner$ and $\epsilon_A = \llcorner 1_A \lrcorner$.

In the following we will see that a name corresponds to a preparation of an entangled quantum state and a coname corresponds to an observational branch resulting from the indeterminism of quantum measurements (Abramsky and Coecke, 2004).

For a morphism $\rho : X \to Y$ in $(\mathcal{R}el, \times)$ we have

$$\ulcorner \rho \urcorner = \{(*, (x, y))|x \rho y, x \in X, y \in Y\},$$
$$\llcorner \rho \lrcorner = \{((x, y), *)|x \rho y, x \in X, y \in Y\}.$$

The compositionality lemma proved in (Abramsky and Coecke, 2004) is the most significant for our argument. It says that the following diagram commutes in any compact closed category:

$$\begin{array}{ccccc} A & \xrightarrow{f} & B & \xrightarrow{g} & C \\ {\scriptstyle r_A}\downarrow & & & & {\scriptstyle l_C^{-1}}\uparrow \\ A \otimes I & \xrightarrow{1_A \otimes \ulcorner g \urcorner} & A \otimes B^* \otimes C & \xrightarrow{\llcorner f \lrcorner \otimes 1_C} & I \otimes C \end{array}$$

The compositionality lemma captures the quantum information flow in the quantum teleportation protocol at an abstract level. The lemma yields the equation

$$U \circ (l_A^{-1} \circ (\llcorner f \lrcorner \otimes 1_A)) \circ ((1_A \otimes \ulcorner g \urcorner) \circ r_A) = U \circ g \circ f,$$

where all morphisms U, g, f have the same domain and codomain A (Fig.2). The original quantum teleportation protocol requires $U \circ g \circ f = 1_A$, however, any composition of morphisms in a compact closed category enjoys the inversion of the order of composition.

The right hand side of the above equation represents the sequence of quantum information flow on one hand, the left hand side represents the causal sequence of quantum measurements and a unitary transformation on the other hand. This distinction between an information flow sequence and a causal sequence is the essential point of our application of the compositionality lemma to biological networks in the following sections.

Algebraic description of network motifs

Network motifs are local patterns in networks that are considered to have certain biological functions (Milo et al., 2002; Alon, 2006, 2007). In particular, a four-nodes network motif called *bi-fan* (Fig.3) is found ubiquitously in information processing biological networks such as gene transcription regulation networks, signal transduction networks and neuronal networks (Milo et al., 2002; Alon, 2006). In this section we explain how the bi-fan motif emerges from an information processing pattern in a network (Haruna, 2008; Haruna and Gunji, 2008).

A node in an information processing network is considered to have an information processing ability. We assume that it has a specific internal structure that represents how

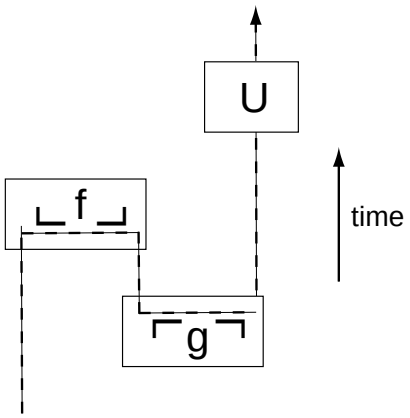

Figure 2: The essential feature of the quantum information fbw can be captured in any compact closed category.

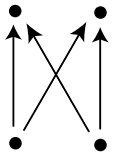

Figure 3: A network motif bi-fan.

it processes information. A simple but non-trivial internal structure considered here consists of two distinct nodes and an arrow between the two nodes:

$$\bullet \rightarrow \bullet$$

The source node, the arrow and the target node are considered to represent reception of information, transformation of information and sending of information, respectively. For example, each node in a gene transcription regulation network is a gene or a protein coded by the gene. They together represent a single node. Hence we can consider information processing in each node: possible regulations from other proteins (reception of information), synthesis of the protein from the gene via the transcription and translation processes (transformation of information) and possible regulations of other genes by the protein (sending of information).

If two nodes with this internal structure are connected by an arrow in the network, this connection by the arrow in the network is represented by a pattern shown below, in which the target in the internal structure of the source node is identifi ed with the source in the internal structure of the target node:

$$\bullet \rightarrow \bullet \rightarrow \bullet$$

Again in gene transcription regulation networks, each arrow in a network indicates a regulation from the source gene to the target gene. The protein synthesized by the source gene

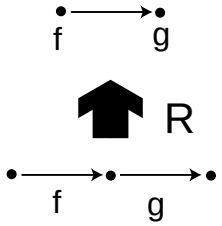

Figure 4: An arrow with its source and target nodes in a network is an image of M by R (See text).

is responsible for the regulation and is included in both sending of information at the source gene and reception of information at the target gene. This motivates us to introduce the above pattern.

We call the pattern *information processing pattern*, which is referred to as M in what follows. Thus an arrow with its source and target nodes in the network can be seen as the image of M by a graph transformation R defi ned as follows (Fig.4).

Let $G = (A_G, O_G, \partial_0^G, \partial_1^G)$ be a directed graph, where A_G is a set of arrows, O_G is a set of nodes and ∂_0^G and ∂_1^G are maps from A_G to O_G. ∂_0^G sends an arrow to its source. ∂_1^G sends an arrow to its target. We defi ne $RG = (A_{RG}, O_{RG}, \partial_0^{RG}, \partial_1^{RG})$ as

$$A_{RG} = \{(f,g) \in A_G^2 | \partial_1^G f = \partial_0^G g\},\ O_{RG} = A_G,$$
$$\partial_0^{RG}(f,g) = f,\ \partial_1^{RG}(f,g) = g.$$

The graph transformation R can be seen as a functor from the category of directed graphs $\mathcal{G}rph$ to itself. The directed graph RG is so called the line graph of G.

In general, we consider the constraint to a local pattern F in a network imposed by the information processing pattern M as that the local pattern F is isomorphic to an image of R, that is, we can write $F \cong RG$ for some G. It can be shown that the condition is equivalent to $\eta_F : F \cong RLF$, where L is a left adjoint to R and η is the unit of the adjunction (Haruna and Gunji, 2008). It is proved that for any information processing pattern M we can construct a corresponding adjoint pair (L, R) (Haruna, 2008). However, the condition $\eta_F : F \cong RLF$ is not equivalent to the condition $F \cong RG$ for some G in general. Here we do not go into the general argument but directly defi ne the left adjoint L.

For a directed graph $G = (A_G, O_G, \partial_0^G, \partial_1^G)$, a directed graph LG consists of the following data:

$$A_{LG} = O_G,\ O_{LG} = (O_G \times \{0,1\})/\sim,$$
$$\partial_0^{LG} x = [(x,0)],\ \partial_1^{LG} x = [(x,1)],$$

where $\sim$ is an equivalence relation generated by a relation ρ defi ned by $(x,1)\rho(y,0) \Leftrightarrow x \rightarrow y$ and $[(x,i)]$ is the equivalence class containing (x,i). We write $x \rightarrow y$ if there is an arrow from x to y in G.

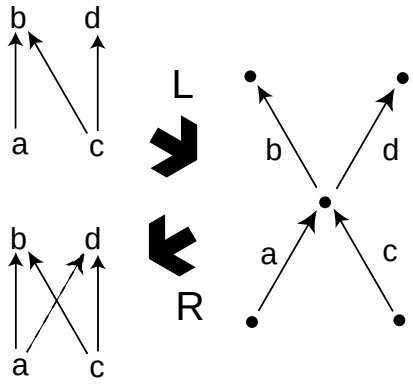

Figure 5: Explanation of the necessary condition for η_F : $F \cong RLF$.

Intuitively, L is a structuration of a pattern by the information processing pattern M because it replaces nodes with arrows. On the other hand, R is a de-structuration of a pattern with respect to M because it collapses an arrow to a node.

The necessary and sufficient condition for $\eta_F : F \cong RLF$ is that F is a binary graph (that is, there is at most one arrow between two nodes) and if $a \to b \leftarrow c \to d$ then $a \to d$ in F. We here explain that the latter condition is necessary. Suppose $a \to b \leftarrow c \to d$ in F (the upper left pattern in Fig.5). If L is performed on this pattern then we obtain the pattern at the right-hand side of Fig.5. If R follows then bi-fan emerges (the lower left pattern in Fig.5) as the dashed arrow is newly added as the fourth arrow.

One can see that $\eta_F : F \cong RLF$ is the condition that the information processing pattern M is fully developed or stabilized in a pattern F. What does happen in this developing process? The key point is what occurs at the central node in the right-hand side pattern in Fig.5. It is an equivalence class consisting of $(a,1),(b,0),(c,1)$ and $(d,0)$. The newly added fourth arrow from a to d appears since $(a,1)$ is identified with $(d,0)$. This identification process is due to the transitive relation $(a,1)\rho(b,0)\rho^{-1}(c,1)\rho(d,0)$. Since $(x,1)\rho(y,0)$ means $x \to y$ in the network level, $(b,0)\rho^{-1}(c,1)$ indicates the existence of feedback from b to c, the direction of which is opposite to the direction of the information flow at the network level.

If we try to interpret the process of emergence of bi-fan described above in terms of the information flows at the network level then an apparent difficulty arises. Since an arrow in information processing biological networks represents the direction of the information flow, it seems that there is no information flow sequence from a to d at the network level. Then how is it possible to construct a connection between a and d? This difficulty arises since the information flows at the network level only cannot treat the above mentioned feedback relation. The difficulty in the interpretation can be resolved when we consider an information flow sequence including the feedback relation and a causal sequence on it.

Reconstruction of causal sequence in biological networks

We reconstruct a causal sequence that brings the emergence of bi-fan through the compositionality lemma by Abramsky and Coecke (2004). We work in the category of sets and relations $(\mathcal{R}el, \times)$ which is compact closed.

We apply the compositionality lemma to the composition $\rho \circ \rho^{-1} \circ \rho$ which brings the fourth arrow in bi-fan. We regard the order of composition represents an information flow sequence including feedback relation ρ^{-1} from b to c, which should be distinguished from the information flows at the network level.

Given a directed graph $F = (A_F, O_F, \partial_0^F, \partial_1^F)$, we put $X = \{(x,0)|x \in O_F\} \cup \{(x,1)|x \in O_F\}$. The compositionality lemma applied to the right-hand side composition of $\rho \circ \rho^{-1} \circ \rho$ gives rise to the following commutative diagram:

$$\begin{array}{ccccc} & & & & X \\ & & & & {\scriptstyle \rho}\uparrow \\ X & \xrightarrow{\rho} & X & \xrightarrow{\rho^{-1}} & X \\ {\scriptstyle r_X}\downarrow & & & & {\scriptstyle l_X^{-1}}\uparrow \\ X \times \{*\} & \xrightarrow{1_X \times \ulcorner \rho^{-1} \urcorner} & X \times X \times X & \xrightarrow{\llcorner \rho \lrcorner \times 1_X} & \{*\} \times X \end{array}$$

The sequence of arrows from the upper left X to the upper right X along the lower side is interpreted as a causal sequence. The feedback relation ρ^{-1} is at the same position as the preparation of entangled qubits pair in the quantum teleportation protocol (Fig.6). The feedback relation ρ^{-1} between b and c is prearranged, so that the information flow from a to d occurs.

In the quantum teleportation case, the causal sequence is the sequence of our operations on the quantum system. A specific causal sequence of our operations enables an 'acausal' quantum information flow to occur. However, in our information processing biological network case, the relation between the information flow sequence and the causal sequence is reversed. We have to reconstruct a causal sequence from a given information flow sequence. Hence there may be an ambiguity in the reconstruction. Indeed, we can also reconstruct a causal sequence in a different way if we apply the compositionality lemma to the left-hand side

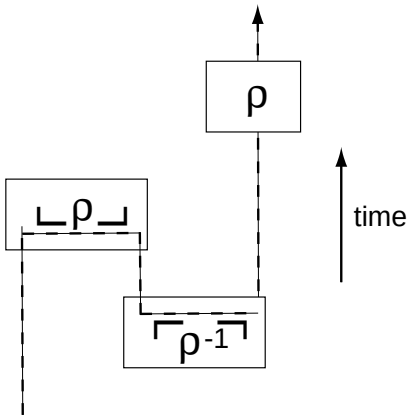

Figure 6: Reconstruction of a causal sequence on the emergence of bi-fan. The information fbw structure is isomorphic to that of the quantum teleportation protocol.

composition of $\rho \circ \rho^{-1} \circ \rho$:

$$
\begin{array}{ccccc}
X & & & & \\
{\scriptstyle \rho}\downarrow & & & & \\
X & \xrightarrow{\rho^{-1}} & X & \xrightarrow{\rho} & X \\
{\scriptstyle r_X}\downarrow & & & & \uparrow{\scriptstyle l_X^{-1}} \\
X \times \{*\} & \xrightarrow{1_X \times \ulcorner \rho \urcorner} & X \times X \times X & \xrightarrow{\llcorner \rho^{-1} \lrcorner \times 1_X} & \{*\} \times X
\end{array}
$$

Contrary to the first reconstruction, the feedback relation ρ^{-1} is at the trail in the corresponding causal sequence (Fig.7). In order to determine a reconstructed causal sequence uniquely, we need a *selection rule*. Since ρ^{-1} is regarded as a biological feedback, it should appear as early as possible in the causal sequence. So the first reconstruction is desired in this respect. If we want to select the first reconstruction, the following rule is sufficient for this example:

ρ^{-1} must be transformed into $\ulcorner \rho^{-1} \urcorner$.

The general argument on how to define a selection rule is beyond the scope of this paper. It is left as a future work.

The difference between $(\mathcal{R}el, \times)$ and the category of finite-dimensional vector spaces over a field should also be noted. Both categories can implement full abstract quantum mechanics with some additional structures on one hand, the former cannot enjoy the full quantum teleportaion protocol since it has no Bell-base consisting of four vectors (Abramsky and Coecke, 2004). Hence the usage of the term 'information fbw' in this paper is different from that in the references (Abramsky and Coecke, 2004; Coecke, 2004). They consider information fbws including the conservation of contents of information. However, we never refer to contents of information but consider only the formal structure among information fbws.

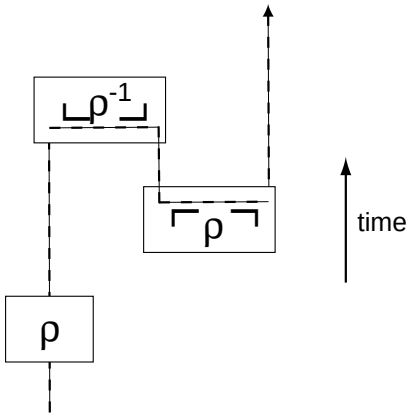

Figure 7: Another reconstruction of a causal sequence on the emergence of bi-fan.

Conclusions

Our argument in this paper is based on a specific example and has not yet developed with full generality. However, we can extract a general strategy to describe changes in biological systems without the explicit external time parameter:

(i) Make a distinction between information fbw sequence and causal sequence.

(ii) Assume selection rules considering what should be prearranged.

(iii) Reconstruct the causal sequence from the information fbw sequence based on the selection rules.

We hope that our argument presented in this paper will help to understand the universal role of information processing in natural phenomena ranging from quantum to biological regimes.

Acknowledgements

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