A PROTOBIOLOGICAL CONSIDERATION ON CELLULAR AUTOMATA

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Abstract

We propose a new construction on cellular automata motivated by protobiology initiated by Koichiro Matsuno. His idea of ambiguity of physics in biological contexts is discussed in the context of ambiguity of symbols. We construct a formal metaphor for ambiguity of state symbols of cellular automata by harnessing the 'up-to-isomorphism ambiguity' of mathematical objects. We show that the introduced ambiguity can enhance the class IV behavior in elementary cellular automata.
Keywords: protobiology, ambiguity, material, cellular automata, class IV.

1. INTRODUCTION

In complex systems research, understanding measurement process inherent in a system is an intriguing problem. Matsuno (1989) studied measurement process in terms of the physics which makes biology possible. According to him, a boundary condition in a biological system cannot be specified completely since it takes finite time to complete measurement process of forces in biological systems like a cell. A physical law that is relevant to a specification of a boundary condition such as the Newton's third law of null-conservation of acting and reacting forces that can be stated by simple symbols (e.g. $F_{i,j} + F_{j,i} = 0$) have ambiguity in the context of more complex material processes typically found in biology than that in physics. The incompleteness of identification of a boundary condition implies that the law of motion is not independent of the identification process of its boundary condition (or measurement). Thus the law of motion cannot be separated from its boundary condition. The law of motion becomes a one-to-many type mapping. One-to-many type mappings provide richer possibilities for future than one-to-one or many-to-one mappings, which might facilitate evolution from simpler objects to more complex organizations. In short, Matsuno presumed that ambiguity of physics in biological contexts can bridge a gap between physics and biology.

Cellular automata (CA) are widely used as a tool for modeling complex systems from physical ones to biological or social ones. A cellular automaton consists of an arrangement of cells on an ordered lattice whose states are in a finite or countable set Ω. The cells interact in parallel with a given local rule f which is defined on a neighborhood of each cell. The Class IV behavior is one of the most intriguing behaviors of one-dimensional cellular automata (Wolfram, 1984). A class IV pattern consists of an erratic mixture of a periodic pattern (class I or II) and a chaotic pattern (class III). The significance of the class IV behavior in natural science can be typically found in modeling the mollusc shell pigmentation. Roughly, there are two different approaches to explain a large variety of patterns of molluscan shells. The first approach is modeling by PDEs (Meinhardt and Klingler, 1987; Meinhardt, 2003). The second one is using CA (Kusch and Markus, 1996). The lack of biochemical knowledge about the mollusc shell pigmentation forces the PDE approach to assume unknown kinetic terms in reaction-diffusion equations. Furthermore, in some shell patterns, the PDE aproach introduces additional independent variables in order to explain complex patterns like branching processes of traveling waves or mixture of different patterns (in CA term, the class IV patterns). In contrast, assumptions about unknown kinetics are not needed in the CA approach since a rule of CA expresses logical relationships of biophysical ideas. a complex pattern formation can be explained by adjusting parameters of CA with a fixed rule. Thus the existence of the class IV patterns in one-dimensional CA provides a smaller explanation of complex pattern formations than switching two mechanisms in PDE approach.

Though the significance of the class IV behavior is demonstrated in modeling shell pattern formation, parameter adjustment is needed in order to generate the class IV patterns. How the class IV behavior arises is not addressed. In this paper, we concern two scenarios how the class IV behavior arises by referring to

Matsuno's idea in more general point of view. Ambiguity of physics in Matsuno is replaced by ambiguity of symbols. In particular, we propose a protobiological formalism on cellular automata. We will show that our formalization of ambiguity of symbols can shed light on the origin of the class IV behavior in one-dimensional cellular automata.

2. A PROTOBIOLOGICAL FORMALISM ON CELLULAR AUTOMATA

In this section, we concern an implementation of Matsuno's idea on cellular automata. First we discuss the idea of ambiguity in more wider context. Second we propose a formalization of the idea.

2.1 Ambiguity of Symbols

Suppose a man performs an addition like $2+3=5$. He might learn the addition through trial and error of examples in his elementary school days. Once he is matured, he can almost always perform the addition without mistakes. This 'almost always' cannot be replaced by 'always'. Though the addition performed by the man can be formally described by axioms (e.g. by Peano axioms), the real addition performed by a man is implemented by material processes in his brain that sometimes makes mistakes. Therefore he is always exposed to the possibility of mistakes. What is implied by the possibility of mistakes? Let us regard that the addition symbol '+' represents some material processes in one's brain. Then the possibility of mistakes means that a symbolization of material processes inevitably have some ambiguity. Once a symbol is fixed, the ambiguity arises as mistakes. Our argument on ambiguity of the addition depends on the fact that a symbol can be combined to different material processes in one's brain, which can sometimes make mistakes. However, we shall presume that the ambiguity is intrinsic to symbols. This is our strategy to avoid falling into symbol-matter dichotomy. Then the possibility of mistakes can be comparable to Matsuno's one-to-many type mapping. Ambiguity of physics is replaced by that of symbols. This replacement enables us to implement Matsuno's idea on cellular automata.

Now let us formalize ambiguity of symbols in the realm of cellular automata. The basic idea is as follows. A mathematical object can be determined uniquely only up to isomorphism. Therefore the algebraic structure of the state space of CA has many isomorphic realizations. Here we fix a larger set than the state space and obtain multiple isomorphic realizations of the algebraic structure by taking appropriate quotients. We shall harness this 'up-to-isomorphism ambiguity' of mathematical objects to construct a metaphor for ambiguity of state symbols in cellular automata.

In order to proceed, we need some lattice-theoretic notions (Davey and Priestley, 2002) since the state space of CA naturally has lattice structure. A *lattice* is a set with two binary operations $\wedge, \vee$ that satisfy commutative, associative and absorption laws. Let us concern a cellular state space $\Omega = \{0,1\}$. We regard Ω as a lattice by introducing two binary operations $\wedge, \vee$ on Ω. $\wedge$ is defined by the following equations; $0\wedge 0 = 0, 0\wedge 1 = 0, 1\wedge 0 = 0$ and $1\wedge 1 = 1$. $\vee$ is defined by the following equations; $0\vee 0 = 0, 0\vee 1 = 1, 1\vee 0 = 1$ and $1\vee 1 = 1$. Let K be a positive integer and Ω^K be the K-fold Cartesian product of Ω. We also make Ω^K into a lattice as follows. For any $x = (x_0, \cdots, x_{K-1}), y = (y_0, \cdots, y_{K-1}) \in \Omega^K$, we define $x\wedge y := (x_0\wedge y_0, \cdots, x_{K-1}\wedge y_{K-1})$ and $x\vee y := (x_0 \vee y_0, \cdots, x_{K-1} \vee y_{K-1})$. A *filter* F of a lattice L is a nonempty subset of L such that for any $x, y \in F$ $x \wedge y \in F$ and for any $x, y \in L$ if $x \leq y$ and $x \in F$ then $y \in F$. Let $F_k (0 \leq k < K)$ be a filter of Ω^K defined by $F_k := \uparrow 1_k$, where $1_k := (x_0, \cdots, x_k, \cdots, x_{K-1})$ with $x_k = 1$ and $x_j = 0 (j \neq k)$, $\uparrow x := \{y \in \Omega^K | x \leq y\}$ for $x \in \Omega^K$ and $x \leq y \overset{\text{def}}{\Leftrightarrow} x \wedge y = x$ for $x, y \in \Omega^K$. A *congruence* θ on a lattice L is an equivalent relation on L that is compatible with both $\wedge$ and $\vee$, that is, if $[x]_\theta = [y]_\theta$ and $[z]_\theta = [w]_\theta$ for any $x, y, z, w \in L$ then $[x \wedge z]_\theta = [y \wedge w]_\theta$ and $[x \vee z]_\theta = [y \vee w]_\theta$ hold, where $[x]_\theta$ is an equivalent class of θ that includes $x \in L$. Let L, M be lattices. A map $f : L \to M$ is called a *lattice isomorphism* if it is bijective and it preserves both $\wedge$ and $\vee$. We have the following lattice isomorphism,

$$\Omega \cong \Omega^K / \theta_k$$

for each $0 \leq k < K$, where $\theta_k := \theta_{F_k} = \{(x, y) \in \Omega^K \times \Omega^K | (\exists z \in F_k) x \wedge z = y \wedge z\}$ is a congruence on Ω^K and Ω^K/θ_k is a quotient lattice induced by the congruence θ_k. Thus the state space Ω has K alternative isomorphic representations given a positive integer K. This ambiguity of the representation of Ω is the key of the following our construction. We have the natural projection associated with θ_k, $\pi_k : \Omega^K \to \Omega^K/\theta_k$ that sends each $x \in \Omega^K$ to the equivalent class which includes x. Since we have the isomorphism (1), we identify π_k with the composition $\phi_k \circ \pi_k$, where ϕ_k is the isomorphism $\phi_k : \Omega^K/\theta_k \to \Omega$ in (1). One can see that the map π_k is determined by the following property; for $x = (x_0, \cdots, x_{K-1}) \in \Omega^K$, if $x_k = 1$ then $\pi_k(x) = 1$ otherwise (when $x_k = 0$) $\pi_k(x) = 0$.

Consider a cell interact with another cell and changes its state by following a intercellular interaction rule $f : \Omega^2 \to \Omega$ between the two cells. The state of cell 0 is denoted by $\omega_0 \in \Omega$ and the state of cell 1 is denoted by $\omega_1 \in \Omega$. Suppose $K = 2$ in equation (1). There exist two congruences θ_0 and θ_1 that satisfy (1). The natural projection associated with $\theta_k (k = 0, 1)$ is a map $\pi_k : \Omega^2 \to \Omega^2/\theta_k \cong \Omega$ that projects an element of Ω^2 to its k-th coordinate. In order to define ambiguity of state symbols $\omega \in \Omega$, we shall associate an element $x_i \in \Omega^2$ and a projection π_{k_i} such that $\pi_{k_i}(x_i) = \omega_i$ with ω_i for $i = 0, 1$. Cell 0 receives an input (ω_0, ω_1) to it as an distribution of x_0, x_1 on Ω^2. Then it observes the distribution by π_0 or π_1 and identifies the input. Let us see some examples. Suppose $\omega_0 = 0, x_0 = (0, 0)$ with $\pi_{k_0} = \pi_0$ and $\omega_1 = 1, x_1 = (0, 1)$ with $\pi_{k_1} = \pi_1$ (Fig. 1). If cell 0 observes x_0, x_1 on Ω^2 by the projection π_0

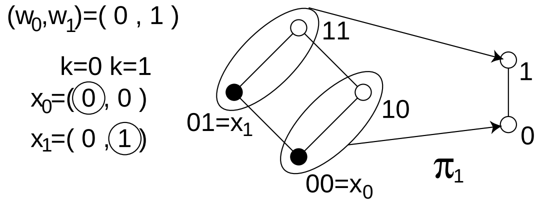

Fig. 1 A correct identification of an input to a cell is possible. A projection π_i that bridges between ω_i and x_i so that $\pi_{k_i}(x_i) = \omega_i$ holds is indicated by a circle on a coordinate of x_i for each $i = 0, 1$. Ω^2 is represented as a Hasse diagram consisting of four nodes at the center of the figure. Ω is also depicted as a Hasse diagram at the right hand side. If cell 0 changes its observation from π_0 to π_1 then it can perform a correct identification of an input $(0, 1)$.

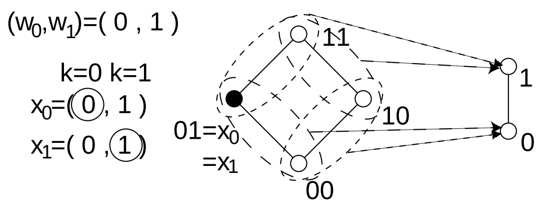

Fig. 2 Any correct identification of an input to a cell is impossible. Cell 0 interprets the input $(0, 1)$ as $(0, 0)$ or $(1, 1)$.

then it cannot identify the input (ω_0, ω_1) correctly since $(\pi_0(x_0), \pi_0(x_1)) = (0, 0) \neq (0, 1) = (\omega_0, \omega_1)$. Hence cell 0 must change its observation in order to perform a correct observation. In this example, the task can be done by its changing its observation from π_0 to π_1. Indeed, we have $\pi_1(x_0) = 0 = \omega_0$ and $\pi_1(x_1) = 1 = \omega_1$. Can a cell always choose a projection by which a correct identification of an input is performed? The answer is negative. There exists an example in which any observation cannot identify an input to a cell correctly. Suppose $\omega_0 = 0, x_0 = (0, 1)$ with $\pi_{k_0} = \pi_0$ and $\omega_1 = 1, x_1 = (0, 1)$ with $\pi_{k_1} = \pi_1$ (Fig. 2). Then we have $(\pi_0(x_0), \pi_0(x_1)) = (0, 0) \neq (0, 1) = (\omega_0, \omega_1)$ and $(\pi_1(x_0), \pi_1(x_1)) = (1, 1) \neq (0, 1) = (\omega_0, \omega_1)$. Cell 0 cannot perform correct identification of the input (ω_0, ω_1) with respect to any projection. Thus the ambiguity of the state space of CA can result in a misidentification of an input to a cell. If the projection π_0 is chosen for π_{k_0} as a result and the intercellular interaction rule f satisfies $f(0, 0) \neq f(0, 1)$ then the transition of the state of cell 0 does not follow the rule f since the input $(\omega_0, \omega_1) = (0, 1)$ is interpreted as $(0, 0)$ by cell 0.

The associated element $x_i \in \Omega^2$ with $\omega_i \in \Omega$ is a virtual factor in terms of an external observer who describes the state of a cell by introducing the state space Ω. The factor introduces an ambiguity of the state. Since we consider that the ambiguity is intrinsic to the state space, the factor can be regarded as a metaphor for that a state symbol is borne by some material processes in the same sense that the addition is implemented by biochemical processes in one's brains. Hence we shall call the factor *material factor*. Our formal definition of protobiological formalism on cellular automata is given in the next subsection.

2.2 Definition of Material Factor-Associated Cellular Automata

In what follows we only concern elementary cellular automata (ECA), that is, one-dimensional binary cellular automata with nearest neighbor interaction (Wolfram, 1983). The formalism can be easily generalized to multi-dimensional multi-valued cellular automata with arbitrary interaction radius.

Let Ω be a lattice $\{0, 1\}$ and K be a positive integer. We define the data of a cell at site i and time t by the triplet $(x_i^t, \omega_i^t, k_i^t)$ with $\pi_{k_i^t}(x_i^t) = \omega_i^t$, where $x_i^t \in \Omega^K, \omega_i^t \in \Omega$ and $0 \leq k_i^t < K$. x_i^t, ω_i^t and $\pi_{k_i^t}$ are called a *material factor*, a *formal state* and an *observation map*, respectively.

Next we describe the rule dynamics in our formalism. Suppose one of 256 rules of ECA $f : \Omega^3 \to \Omega$ is given. A cell at site i ant time t receives an input $(\omega_{i-1}^t, \omega_i^t, \omega_{i+1}^t) \in \Omega^3$ to it as a distribution of material factors $x_{i-1}^t, x_i^t, x_{i+1}^t$ on Ω^K. The number of possible observations is K. As we have seen in the previous subsection, there exists a situation that a cell cannot perform a correct observation of an input in principle. Since we external observers describe the behavior of cells by introducing the state space Ω, the degree of misidentification of an input should be minimized in the actual realized observation. Motivated by this consideration, we define the time evolution of cellular data as follows; define

$$\sigma_{i,k}^t := \sum_{j=i-1,i,i+1} d(\omega_j^t, \pi_k(x_j^t)),$$

$$\delta_i^t := \min_{0 \leq k < K} \sigma_{i,k}^t,$$

and

$$S_i^t := \{k | \sigma_{i,k}^t = \delta_i^t\},$$

where d is a metric on Ω given by setting $d(0, 1) = 1$. The update of the data of a cell at site i and time t is performed by the following three steps.

(i) Choosing an observation. An element of S_i^t is chosen by following the equiprobability distribution on S_i^t. We write the chosen element k_i^{t+1}.

(ii) Updating material factor x_i^t. For $k \neq k_i^{t+1}$, $\pi_k(x_i^{t+1}) = \pi_k(x_i^t)$. For $k = k_i^{t+1}$,

$$\pi_{k_i^{t+1}}(x_i^{t+1}) = f((\pi_{k_i^{t+1}}(x_j^t))_{j=i-1,i,i+1}).$$

These equations determines x_i^{t+1} uniquely since $x_i^{t+1} = (\pi_0(x_i^{t+1}), \cdots, \pi_{K-1}(x_i^{t+1}))$.

(iii)Updating formal state ω_i^t.

$$\omega_i^{t+1} = \pi_{k_i^{t+1}}(x_i^{t+1}).$$

The above three steps defines the time evolution of the cellular data from $(x_i^t, \omega_i^t, k_i^t)$ to $(x_i^{t+1}, \omega_i^{t+1}, k_i^{t+1})$. We call the time evolutionary system defined above *material factor-associated cellular automata (MFCA)*. Note that if $K = 1$ then any MFCA is just a usual ECA.

3. BEHAVIORS OF MATERIAL FACTOR-ASSOCIATED CELLULAR AUTOMATA

In this section first we observe behaviors of some examples of MFCA in order to find a clue as to how mixture of a periodic pattern and a chaotic pattern (the class IV behavior) can arise in MFCA. Second we apply a class discrimination method proposed by Kusch and Markus (1996) with a modification to MFCA in order to see how many rules show the class IV behavior under the MFCA construction.

3.1 Examples of Patterns

We show space-time patterns of four examples of MFCA compared with those of ECA in Fig. 3. The number of a rule is specified by following Wolfram (1984). The system size is 100 with the periodic boundary condition. The cells are arranged along the horizontal line and evolve their state along the vertical direction from the top to the bottom. The first 150 time steps are shown initiated from the random initial condition. The random initial condition here means as follows. For each cell at site i, first its initial formal state ω_i^0 is chosen to be 0 or 1 with probability $\frac{1}{2}$. Second its initial observation $\pi_{k_i^0}$ is chosen from the set of all projections $\{\pi_0, \cdots, \pi_{K-1}\}$ with equiprobability. Then a coordinate of its initial material factor $\pi_{k_i^0}(x_i^0)$ is automatically determined by $\pi_{k_i^0}(x_i^0) := \omega_i^0$. Finally the value of $\pi_k(x_i^0)$ is chosen to be 0 or 1 with probability $\frac{1}{2}$ for $k \neq k_i^0$. Thus an initial condition for a site is specified.

For each rule, the left hand side picture shows a usual ECA time evolution of the rule. Rule 1 (Fig. 3(a)) and rule 7 (Fig. 3(b)) are class II (periodic patterns) rules. Rule 22 (Fig. 3(c)) and rule 90 (Fig. 3(d)) are classified in to class III (chaotic patterns). This corresponds to the case $K = 1$ in MFCA. Cells with state 0 and 1 are represented by white and black squares, respectively. The picture at the center shows a time evolution of the rule under the MFCA construction with $K = 10$ for rule 1 (Fig. 3(a)), $K = 8$ for rule 7 (Fig. 3(b)), $K = 3$ for rule 22 (Fig. 3(c)) and $K = 5$ for rule 90 (Fig. 3(d)). Space-time patterns in the formal states of the cells are depicted. Cells with formal state 0 and 1 are represented by white and black squares, respectively. They show the class IV behavior which is an erratic mixture of a periodic pattern and a chaotic pattern. In the right hand side pictures, if $\omega_i^t \neq f(\omega_{i-1}^t, \omega_i^t, \omega_{i+1}^t)$ holds in the pictures at the center then a cell at site i

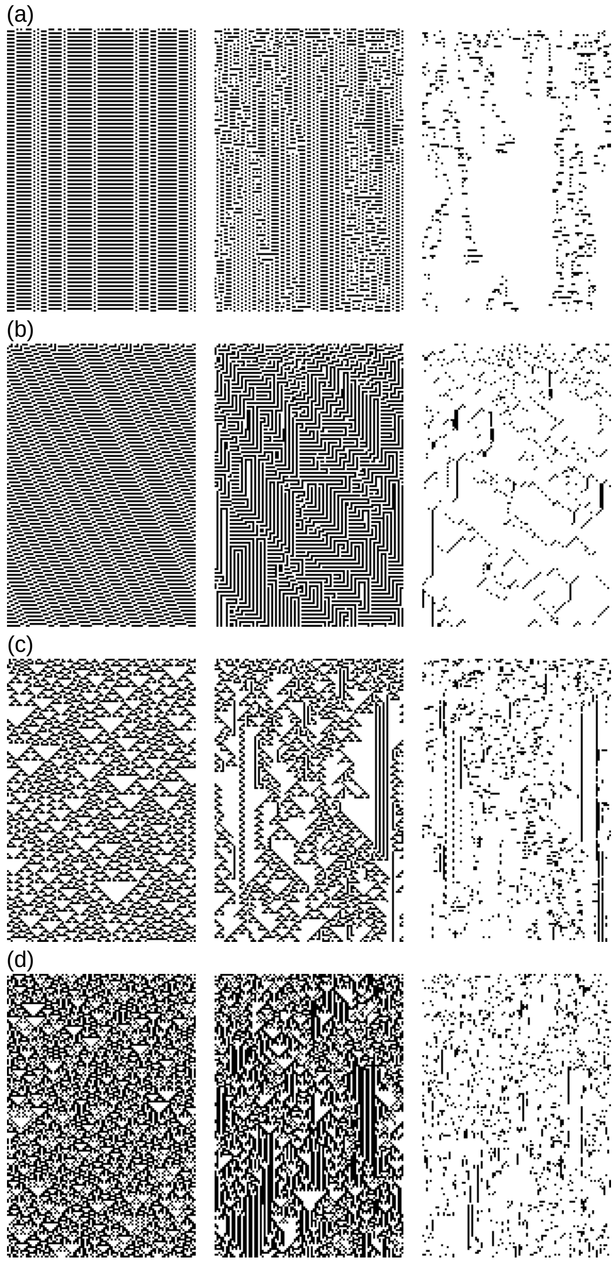

Fig. 3 Space-time patterns of four ECA rules under the MFCA construction. The system size is 100 with the periodic boundary condition. The first 150 time steps are shown. The initial arrangements are given randomly as explained in the text. Left hand side pictures are space-time patterns with $K = 1$ (usual ECA patterns). Pictures at the center are space-time patterns under the MFCA construction with some $K > 1$. The right hand side picture indicates when and where miscalculations of the formal state of a cell occur. (a)Rule 1 with $K = 10$. (b)Rule 7 with $K = 8$. (c)Rule 22 with $K = 3$. (d)Rule 90 with $K = 5$.

and time t is depicted by a black square otherwise the cell is depicted by a white square.

In Fig. 3 (a) and (b), class II patterns are made into class IV patterns with erratic miscalculation propagation by the MFCA construction. In contrast, class III rules can generate class IV patterns under the MFCA construction as shown in Fig. 3 (c) and (d). From these observations, we can suggest two scenarios of how the class IV behavior can arise in the MFCA construction. The first scenario is that miscalculations of formal state values erratically propagate against class II patterns background (Fig. 3 (a) and (b)). The second scenario is that rare arrangements in typical class III ECA space-time patterns are generated locally and remain certain periods sometimes bounded by walls of miscalculations (Fig. 3 (c) and (d)). For example, the periodic arrangement $\cdots 110110110 \cdots$ is a fix point of rule 90 ($100, 110, 001, 110 \rightarrow 1$ otherwise 0) in infinite one-dimensional lattice. However, this periodic arrangement cannot be observed in typical time evolutions of rule 90 from the random initial conditions. If a finite repetition of 110 is generated, it disappears immediately (Fig. 3 (d), the left hand side picture). In contrast, walls of miscalculations can help a finite repetition of 110 remain in certain periods in rule 90 under the MFCA construction with $K = 5$ (Fig. 3 (d), the pictures at the center and the right hand side).

The intimate analysis of mechanisms of the class IV behavior in the MFCA construction remains as an open task. In the next subsection, we observe in what degree the class IV behavior is enforced under the MFCA construction.

3.2 Discriminating the Different Classes

Kusch and Markus (1996) introduce a new method to discriminate space-time patterns of CA. Their idea is to quantify what we observe when we see difference-patterns (DP). A DP is the difference between a pattern with a perturbation to a single cell at initial time $t = 0$ and a pattern without any perturbation. The difference between class III and class IV appears clearly in the DPs (Fig. 4). The two borders of a DP for class III CA propagate with a constant velocity to the right and a constant velocity to the left (Fig. 4 (c)). On the other hand, the borders of a DP for class IV CA changes their propagation velocities erratically (Fig. 4 (b) and (d)). In order to quantify these characteristics, Kusch and Markus calculate the standard deviation Δm of the slope of a fitted line for each border of a DP for symmetric CA. Before calculating the standard deviation Δm, they normalize slope of a fitted line. They suggest the following classification:

(i) In class I (homogeneous patterns), DPs vanish after short periods.

(ii) Δm for class II (periodic patterns) are nearly zero since the perturbations are confined in a finite space. Hence the borders can be fitted without large errors.

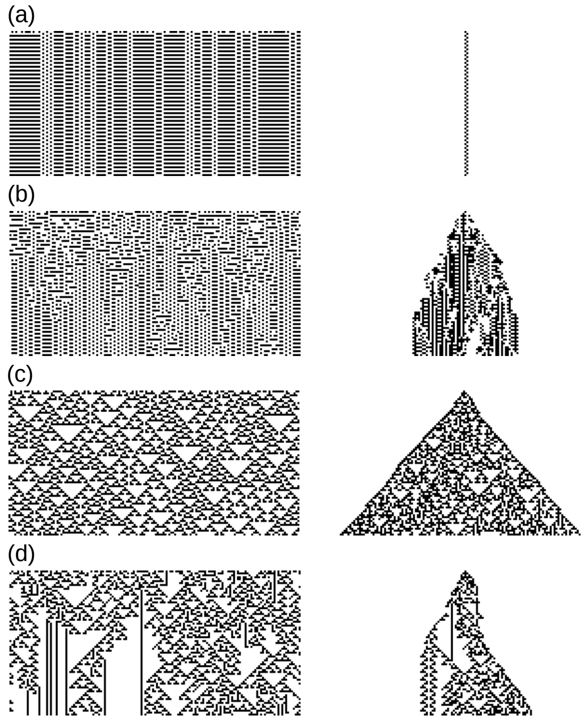

Fig. 4 Examples of difference-patterns (DP). Two run of a CA is performed in parallel from the same initial arrangement except the cell at the center. We perturb the material factor of the cell at the center by flipping its all coordinates. (a)Rule 1 with $K = 1$ (class II). (b)Rule 1 with $K = 10$ (class IV). (c)Rule 22 with $K = 1$ (class III). (d)Rule 22 with $K = 3$ (class IV).

(iii) For class III (chaotic patterns), Δm is small relative to that of class IV since perturbations propagate with constant velocities with small-scale fluctuations.

(iv) Since class IV (co-existing of both periodic and chaotic patterns) DPs have erratic mixture of oblique line borders and vertical line borders, Δm is larger than the other classes.

Since Kusch and Markus treat only symmetric rules, they need not to distinguish Δm for the left border from that for the right border. However, there are many asymmetric rules in all 256 ECA rules. Hence we make a distinction between Δm_l for the left border and Δm_r for the right border.

The pair $(\Delta m_r, \Delta m_l)$ for each ECA rule is plotted in Fig. 5. The rules that shows the class IV behavior are clustered around $(0.004, 0.004)$. The rules that can be classified into the other classes have smaller Δm values ($\lesssim 0.002$) than those in class IV. A small fraction of rules have large Δm_r or Δm_l if $K = 1$ (Fig. 5 (a)). In contrast, much larger number of rules show large Δm values (Fig. 5 (b), (c) and (d)). Note that this plot contains some exceptional rules. DPs for some rules (e.g. rule 43) show zigzag lines. These rules are depicted with considerably high Δm values ($\gtrsim 0.008$) or with

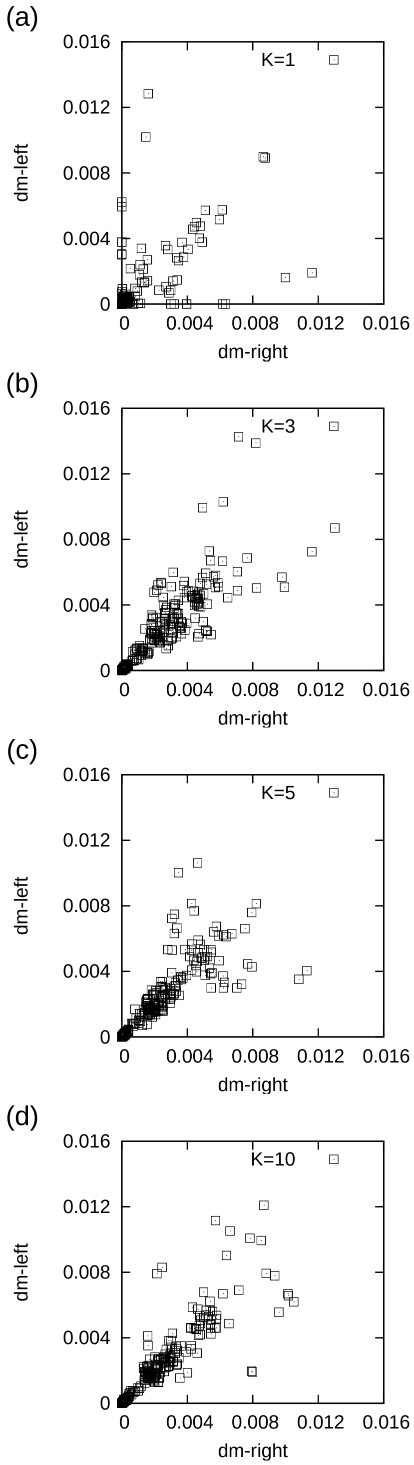

Fig. 5 $(\Delta m_r, \Delta m_l)$ plots for all 256 ECA rules. For each rule, Δm_r and Δm_l are averaged over 100 different DPs. The system size is 501 with the periodic boundary condition. DPs are determined by first 250 time steps. (a)$K = 1$. (b)$K = 3$. (c)$K = 5$. (d)$K = 10$. The number of rules with high Δm values under the MFCA construction with $K > 1$ is much larger than with $K = 1$.

large displacement from the diagonal line. Though they cannot be classified into class IV, they have large Δm values. In spite of the existence of such small number of exceptions, the $(\Delta m_r, \Delta m_l)$ plots for all ECA rules suggest that the MFCA construction can enhance the class IV behavior for $K > 1$.

4. CONCLUDING REMARKS

In this paper, we propose a formalization of the idea of ambiguity of symbols in cellular automata. The 'up-to-isomorphism ambiguity' $\Omega^K/\theta_k \cong \Omega$ of the cellular state space serves as our MFCA construction. We presume that the ambiguity is intrinsic to the state space Ω so that we can use the ambiguity as a metaphor for material. An element of the lattice Ω^K is called material factor. Material factors can lead to a miscalculation of a cellular state transition that facilitate the emergence of the class IV behavior in ECA under the MFCA construction.

We only concern finite K in this paper. However, one can construct MFCA with $K = \infty$ as with the finite K case. That is, we turn our eyes to the isomorphisms $\Omega^{\mathbb{N}}/\theta_k \cong \Omega$, where $\mathbb{N}$ is the set of natural numbers. Then the time evolutions of the formal states in the obtained MFCA from the random initial conditions contain no miscalculation since at any site and any time there exist infinitely many correct observations of any input to a cell. The finiteness of Ω^K is the essential factor in order that the material factor can work explicitly.

One can give the necessary and sufficient condition for the existence of a miscalculation when $K > 1$. In the realm of ECA, there exist $\boldsymbol{\omega}, \boldsymbol{\omega'} \in \Omega^3$ such that $f(\boldsymbol{\omega}) = 0$ and $f(\boldsymbol{\omega'}) = 1$ if and only if we can find formal states, material factors and observations such that a miscalculation can occur. The proof will be given elsewhere in more general form.

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